Summer 2016

Norwalk Early College Academy

Honors Algebra 2
SUMMER PACKET

In order to be successful in Algebra 2, you must have certain prerequisite skills mastered. You will be assessed on the content of this packet during the first week of school.

Please make your best effort as you work on this packet. You can work with another person, but keep in mind that each person has to take the quiz. Please show all of your work.

Enjoy your summer!! I look forward to meeting you and working with you when return to school in the fall.

Ms. Quermorllue

Summer 2016

Algebra 1 Skills Needed to be Successful in Algebra 2

A. Simplifying Polynomial Expressions

Objectives: The student will be able to:

- Apply the appropriate arithmetic operations and algebraic properties needed to simplify an algebraic expression.
- Simplify polynomial expressions using addition and subtraction.
- Multiply a monomial and polynomial.

B. Solving Equations

Objectives: The student will be able to:

- Solve multi-step equations.
- Solve a literal equation for a specific variable, and use formulas to solve problems.

C. Rules of Exponents

Objectives: The student will be able to:

- Simplify expressions using the laws of exponents.
- Evaluate powers that have zero or negative exponents.

D. Binomial Multiplication

Objectives: The student will be able to:

- Multiply two binomials.

E. Factoring

Objectives: The student will be able to:

- Identify the greatest common factor of the terms of a polynomial expression.
- Express a polynomial as a product of a monomial and a polynomial.
- Find all factors of the quadratic expression $ax^2 + bx + c$ by factoring and graphing.

F. Radicals

Objectives: The student will be able to:

- Simplify radical expressions.

G. Graphing Lines

Objectives: The student will be able to:

- Identify and calculate the slope of a line.
- Graph linear equations using a variety of methods.
- Determine the equation of a line.

A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "like", or terms that have the same variable(s) with the same exponent(s).

Ex. 1: $5x - 7y + 10x + 3y$

$\underline{5x} \underline{- 7y} + \underline{10x} + \underline{3y}$

$15x - 4y$

Ex. 2: $-8h^2 + 10h^3 - 12h^2 - 15h^3$

$\underline{-8h^2} + \underline{10h^3} \underline{- 12h^2} \underline{- 15h^3}$

$-20h^2 - 5h^3$

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

Ex. 1: $3(9x-4)$

$3 \cdot 9x - 3 \cdot 4$

$27x - 12$

Ex. 2: $4x^2(5x^3+6x)$

$4x^2 \cdot 5x^3 + 4x^2 \cdot 6x$

$20x^5 + 24x^3$

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

Ex. 1: $3(4x-2)+13x$

$3 \cdot 4x - 3 \cdot 2 + 13x$

$12x - 6 + 13x$

$25x - 6$

Ex. 2: $3(12x-5)-9(-7+10x)$

$3 \cdot 12x - 3 \cdot 5 - 9(-7) - 9(10x)$

$36x - 15 + 63 - 90x$

$-54x + 48$

PRACTICE SET 1

Simplify.

1. $8x - 9y + 16x + 12y$

2. $14y + 22 - 15y^2 + 23y$

3. $5n - (3 - 4n)$

4. $-2(11b - 3)$

5. $10q(16x + 11)$

6. $-(5x - 6)$

7. $3(18z - 4w) + 2(10z - 6w)$

8. $(8c + 3) + 12(4c - 10)$

9. $9(6x - 2) - 3(9x^2 - 3)$

10. $-(y - x) + 6(5x + 7)$

B. Solving Equations

I. Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations and work in the reverse order.
2. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

$$\begin{aligned} Ex.1:\ 4x-2&=30 \\ +2\ \ &\ +2 \\ 4x&=32 \\ \div 4\ \ &\ \div 4 \\ x&=8 \end{aligned}$$

$$\begin{aligned} Ex.2:\ 87&=-11x+21 \\ -21\ \ &\qquad\ -21 \\ 66&=-11x \\ \div -11\ \ &\ \div -11 \\ -6&=x \end{aligned}$$

II. Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

$$\begin{aligned} Ex.3:\ 8x+4&=4x+28 \\ -4\ \ &\qquad\ -4 \\ 8x&=4x+24 \\ -4x\ \ &\ -4x \\ 4x&=24 \\ \div 4\ \ &\ \div 4 \\ x&=6 \end{aligned}$$

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

$$\begin{aligned} Ex.4:\ 5(4x-7)&=8x+45+2x \\ 20x-35&=10x+45 \\ -10x\qquad&\ -10x \\ 10x-35&=45 \\ +35\ \ &\ +35 \\ 10x&=80 \\ \div 10\ \ &\ \div 10 \\ x&=8 \end{aligned}$$

PRACTICE SET 2

Solve each equation. You must show all work.

1. $5x - 2 = 33$
2. $140 = 4x + 36$
3. $8(3x - 4) = 196$
4. $45x - 720 + 15x = 60$
5. $132 = 4(12x - 9)$
6. $198 = 154 + 7x - 68$
7. $-131 = -5(3x - 8) + 6x$
8. $-7x - 10 = 18 + 3x$
9. $12x + 8 - 15 = -2(3x - 82)$
10. $-(12x - 6) = 12x + 6$

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

$$Ex.\,1:\ 3xy = 18, \quad \text{Solve for } x.$$

$$\frac{3xy}{3y} = \frac{18}{3y}$$

$$x = \frac{6}{y}$$

$$Ex.\,2:\ 5a - 10b = 20, \quad \text{Solve for } a.$$

$$+10b = +10b$$

$$5a = 20 + 10b$$

$$\frac{5a}{5} = \frac{20}{5} + \frac{10b}{5}$$

$$a = 4 + 2b$$

PRACTICE SET 3

Solve each equation for the specified variable.

1. $Y + V = W$, for V

2. $9wr = 81$, for w

3. $2d - 3f = 9$, for f

4. $dx + t = 10$, for x

5. $P = (g - 9)180$, for g

6. $4x + y - 5h = 10y + u$, for x

C. Rules of Exponents

Multiplication: Recall $(x^m)(x^n)=x^{(m+n)}$ $\quad$ *Ex*: $(3x^4y^2)(4xy^5)=(3\cdot 4)(x^4\cdot x^1)(y^2\cdot y^5)=12x^5y^7$

Division: Recall $\dfrac{x^m}{x^n}=x^{(m-n)}$ $\quad$ *Ex*: $\dfrac{42m^5j^2}{-3m^3j}=\left(\dfrac{42}{-3}\right)\left(\dfrac{m^5}{m^3}\right)\left(\dfrac{j^2}{j^1}\right)=-14m^2j$

Powers: Recall $(x^m)^n = x^{(m\cdot n)}$ $\quad$ *Ex*: $(-2a^3bc^4)^3=(-2)^3(a^3)^3(b^1)^3(c^4)^3=-8a^9b^3c^{12}$

Power of Zero: Recall $x^0=1,\ x\neq 0$ $\quad$ *Ex*: $5x^0y^4=(5)(1)(y^4)=5y^4$

PRACTICE SET 4

Simplify each expression.

1. $(c^5)(c)(c^2)$
2. $\dfrac{m^{15}}{m^3}$
3. $(k^4)^5$
4. d^0
5. $(p^4q^2)(p^7q^5)$
6. $\dfrac{45y^3z^{10}}{5y^3z}$
7. $(-t^7)^3$
8. $3f^3g^0$
9. $(4h^5k^3)(15k^2h^3)$
10. $\dfrac{12a^4b^6}{36ab^2c}$
11. $(3m^2n)^4$
12. $(12x^2y)^0$
13. $(-5a^2b)(2ab^2c)(-3b)$
14. $4x(2x^2y)^0$
15. $(3x^4y)(2y^2)^3$

D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

$$Ex\ 1:\ 8(5x^2 - 9x)$$
$$8 \cdot 5x^2 + 8 \cdot (-9x)$$
$$40x^2 - 72x$$

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with <u>two</u> terms), we use the "FOIL" method. The "FOIL" method uses the distributive property <u>twice</u>!

FOIL is the order in which you will multiply your terms.

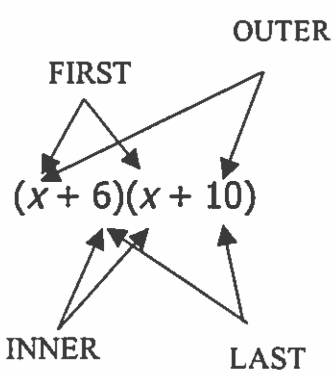
First

Outer

Inner

Last

Ex. 1: $(x + 6)(x + 10)$



First	$x \cdot x$ ------> x^2
Outer	$x \cdot 10$ -----> $10x$
Inner	$6 \cdot x$ ------> $6x$
Last	$6 \cdot 10$ -----> 60

$$x^2 + 10x + 6x + 60$$

$$x^2 + 16x + 60$$
(After combining like terms)

Recall: $4^2 = 4 \cdot 4$

$x^2 = x \cdot x$

Ex. $(x + 5)^2$

$(x + 5)^2 = (x + 5)(x+5)$

Now you can use the "FOIL" method to get a simplified expression.

PRACTICE SET 5

Multiply. Write your answer in simplest form.

1. $(x + 10)(x - 9)$
2. $(x + 7)(x - 12)$
3. $(x - 10)(x - 2)$
4. $(x - 8)(x + 81)$
5. $(2x - 1)(4x + 3)$
6. $(-2x + 10)(-9x + 5)$
7. $(-3x - 4)(2x + 4)$
8. $(x + 10)^2$
9. $(-x + 5)^2$
10. $(2x - 3)^2$

E. Factoring

I. Using the Greatest Common Factor (GCF) to Factor.

- Always determine whether there is a greatest common factor (GCF) first.

Ex. 1 $3x^4 - 33x^3 + 90x^2$

  - In this example the GCF is $3x^2$.
  - So when we factor, we have $3x^2(x^2 - 11x + 30)$.
  - Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

30		30	
1	30	-1	-30
2	15	-2	-15
3	10	-3	-10
5	6	-5	-6

Since -5 + -6 = -11 and (-5)(-6) = 30 we should choose -5 and -6 in order to factor the expression.

  - The expression factors into $3x^2(x-5)(x-6)$

Note: Not all expressions will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Ex. 2 $4x^3 - 100x$

$4x(x^2 - 25)$

$4x(x-5)(x+5)$

Since x^2 and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.

PRACTICE SET 6

Factor each expression.

1. $3x^2 + 6x$

2. $4a^2b^2 - 16ab^3 + 8ab^2c$

3. $x^2 - 25$

4. $n^2 + 8n + 15$

5. $g^2 - 9g + 20$

6. $d^2 + 3d - 28$

7. $z^2 - 7z - 30$

8. $m^2 + 18m + 81$

9. $4y^3 - 36y$

10. $5k^2 + 30k - 135$

F. Radicals

To simplify a radical, we need to find the greatest perfect square factor of the number under the radical sign (the radicand) and then take the square root of that number.

$Ex. 1: \sqrt{72}$

$\sqrt{36} \cdot \sqrt{2}$

$6\sqrt{2}$

$Ex. 2: 4\sqrt{90}$

$4 \cdot \sqrt{9} \cdot \sqrt{10}$

$4 \cdot 3 \cdot \sqrt{10}$

$12\sqrt{10}$

$Ex. 3: \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

OR

$Ex. 3: \sqrt{48}$

$\sqrt{4}\sqrt{12}$

$2\sqrt{12}$

$2\sqrt{4}\sqrt{3}$

$2 \cdot 2 \cdot \sqrt{3}$

$4\sqrt{3}$

This is not simplified completely because 12 is divisible by 4 (another perfect square)

<u>PRACTICE SET 7</u>

Simplify each radical.

1. $\sqrt{121}$
2. $\sqrt{90}$
3. $\sqrt{175}$
4. $\sqrt{288}$
5. $\sqrt{486}$
6. $2\sqrt{16}$
7. $6\sqrt{500}$
8. $3\sqrt{147}$
9. $8\sqrt{475}$
10. $\sqrt{\dfrac{125}{9}}$

G. Graphing Lines

I. Finding the Slope of the Line that Contains each Pair of Points.

Given two points with coordinates (x_1, y_1) and (x_2, y_2), the formula for the slope, m, of the line containing the points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Ex. (2, 5) and (4, 1)

$$m = \frac{1-5}{4-2} = \frac{-4}{2} = -2$$

The slope is -2.

Ex. (-3, 2) and (2, 3)

$$m = \frac{3-2}{2-(-3)} = \frac{1}{5}$$

The slope is $\frac{1}{5}$

PRACTICE SET 8

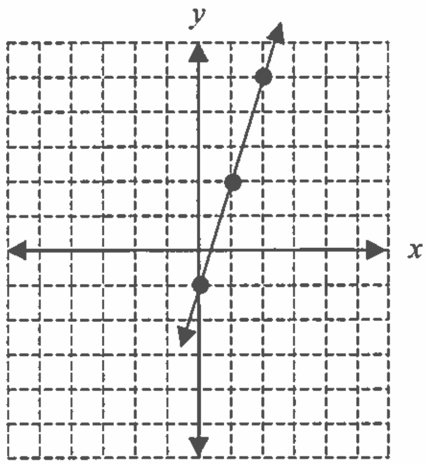
1. (-1, 4) and (1, -2)
2. (3, 5) and (-3, 1)
3. (1, -3) and (-1, -2)
4. (2, -4) and (6, -4)
5. (2, 1) and (-2, -3)
6. (5, -2) and (5, 7)

II. Using the Slope – Intercept Form of the Equation of a Line.

The slope-intercept form for the equation of a line with slope m and y-intercept b is $y = mx + b$.

Ex. $y = 3x - 1$

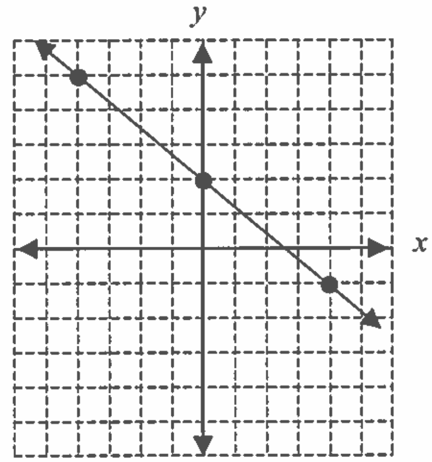
Slope: 3 y-intercept: -1



Place a point on the y-axis at -1. Slope is 3 or 3/1, so travel up 3 on the y-axis and over 1 to the right.

Ex. $y = -\frac{3}{4}x + 2$

Slope: $-\frac{3}{4}$ y-intercept: 2

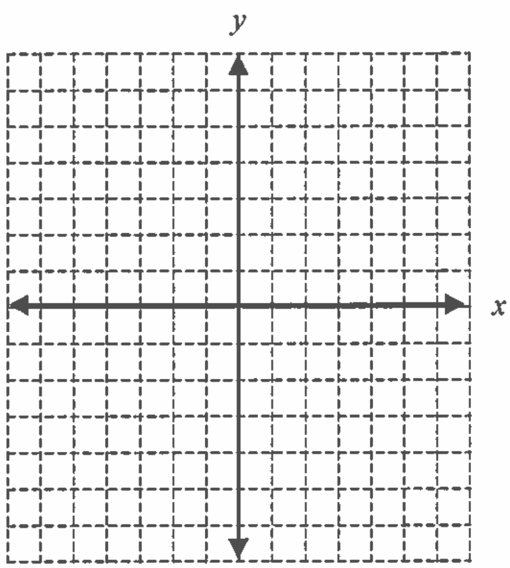


Place a point on the y-axis at 2. Slope is -3/4 so travel down 3 on the y-axis and over 4 to the right. Or travel up 3 on the y-axis and over 4 to the left.

PRACTICE SET 9

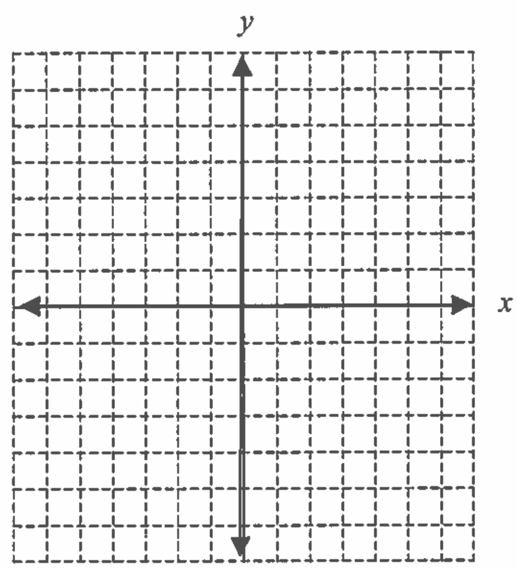
1. $y = 2x + 5$

Slope: _____ y-intercept: _____



2. $y = \frac{1}{2}x - 3$

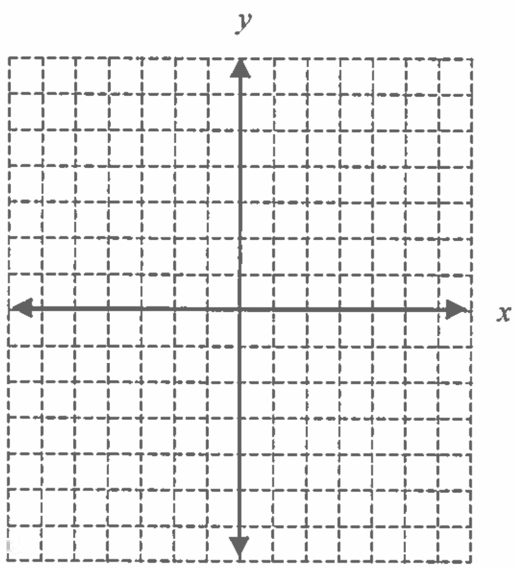
Slope: _____ y-intercept: _____



3. $y = -\frac{2}{5}x + 4$

Slope: ______________

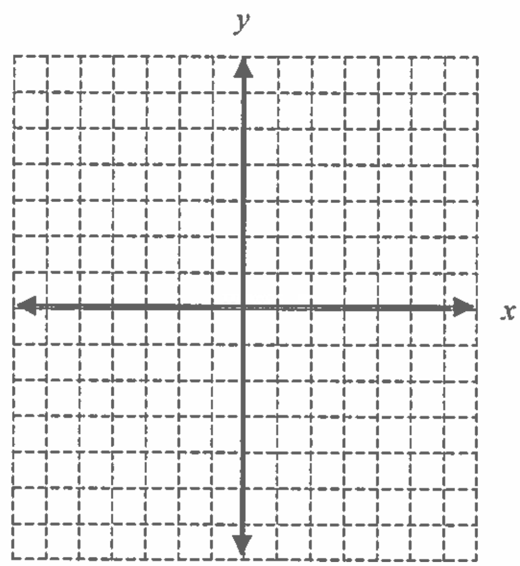
y-intercept: ______________



4. $y = -3x$

Slope: ______________

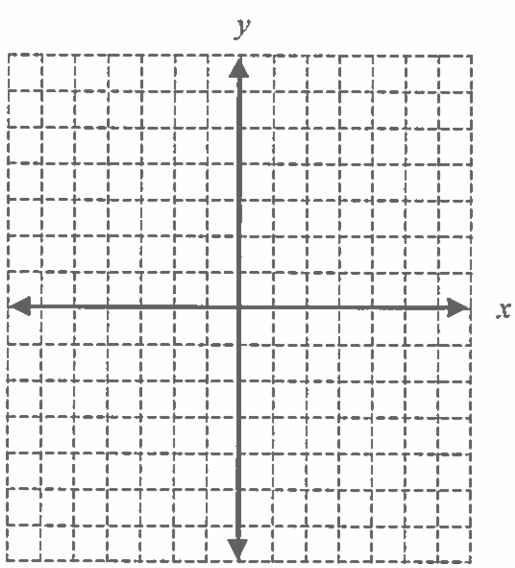
y-intercept ______________



5. $y = -x + 2$

Slope: ______________

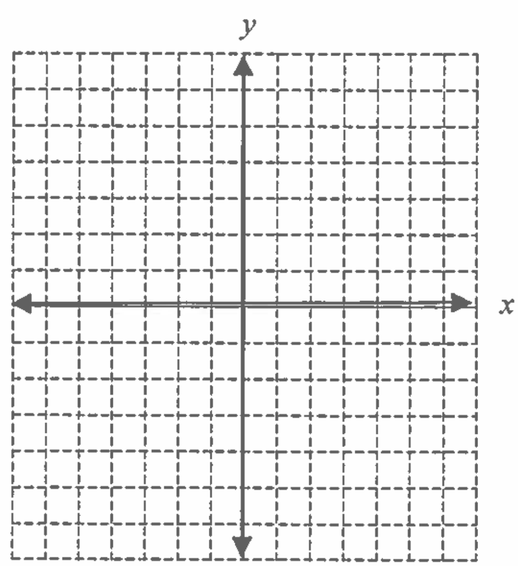
y-intercept: ______________



6. $y = x$

Slope: ______________

y-intercept ______________



III. Using Standard Form to Graph a Line.

An equation in standard form can be graphed using several different methods. Two methods are explained below.

a. Re-write the equation in $y = mx + b$ form, identify the y-intercept and slope, then graph as in Part II above.

b. Solve for the x- and y- intercepts. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y. Then plot these points on the appropriate axes and connect them with a line.

Ex. $2x - 3y = 10$

a. Solve for y.

$$-3y = -2x + 10$$

$$y = \frac{-2x+10}{-3}$$

$$y = \frac{2}{3}x - \frac{10}{3}$$

OR

b. Find the intercepts:

let $y = 0$:

$$2x - 3(0) = 10$$

$$2x = 10$$

$$x = 5$$

So x-intercept is (5, 0)

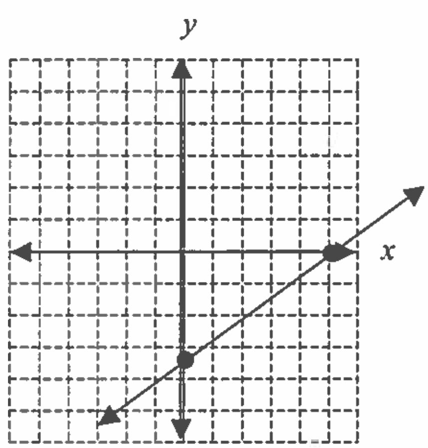
let $x = 0$:

$$2(0) - 3y = 10$$

$$-3y = 10$$

$$y = -\frac{10}{3}$$

So y-intercept is $\left(0, -\frac{10}{3}\right)$



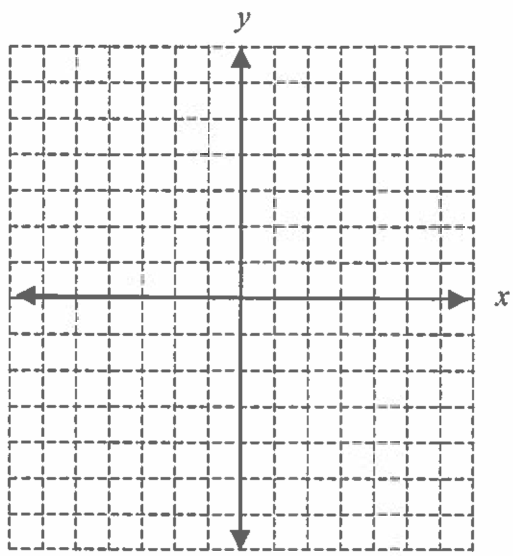
On the x-axis place a point at 5.

On the y-axis place a point at $-\frac{10}{3} = -3\frac{1}{3}$

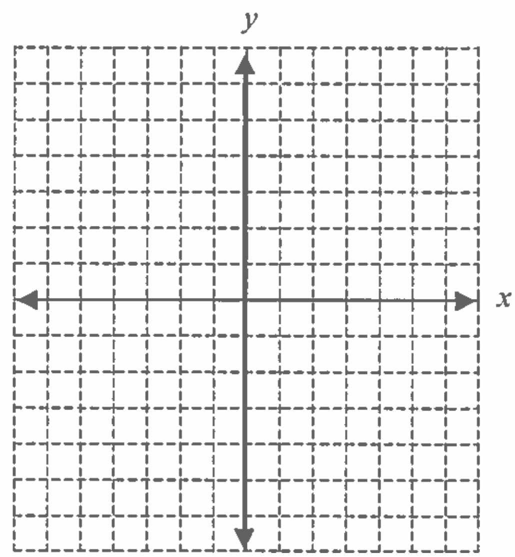
Connect the points with the line.

PRACTICE SET 10

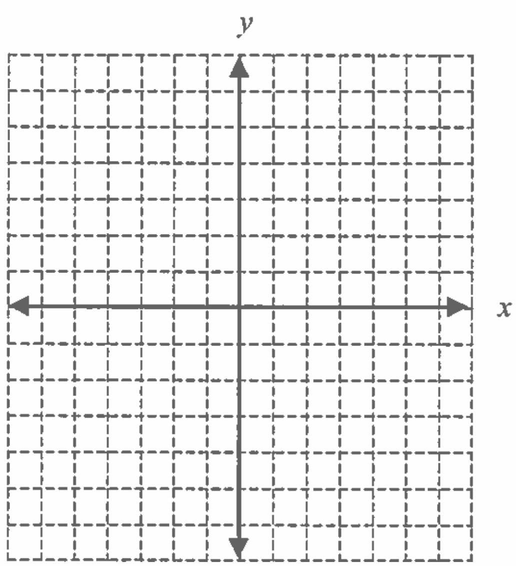
1. $3x + y = 3$



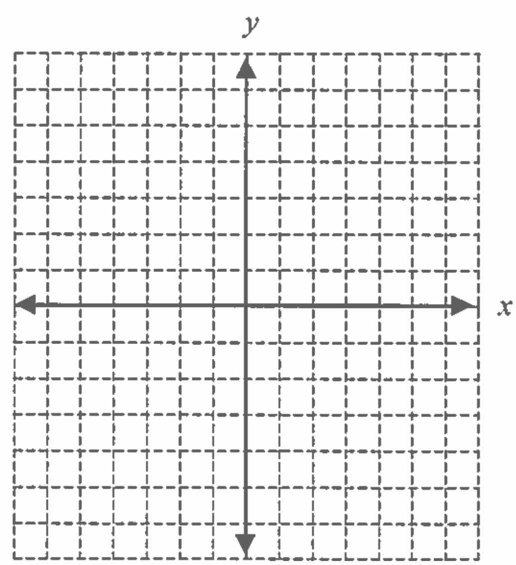
2. $5x + 2y = 10$



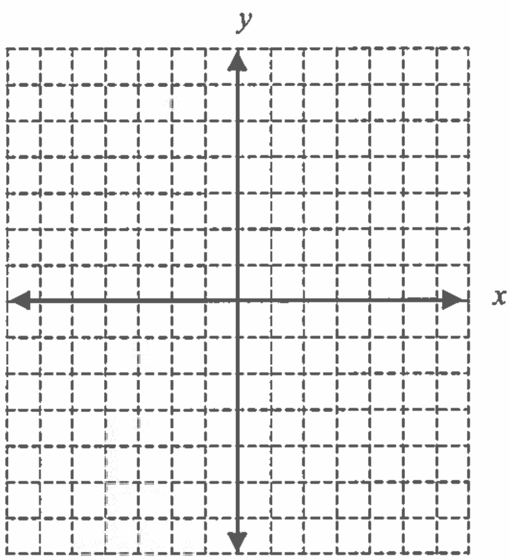
3. $y = 4$



4. $4x - 3y = 9$



5. $-2x + 6y = 12$



6. $x = -3$